Efficient Monadic Streams

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FELDSPAR

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Functional Embedded Language for
Digital Signal Processing and Parallelism

FELDSPAR

- Embedded in Haskell
- Generates C
- Aims to raise the level of abstraction for the programming

- Streams for signal processing

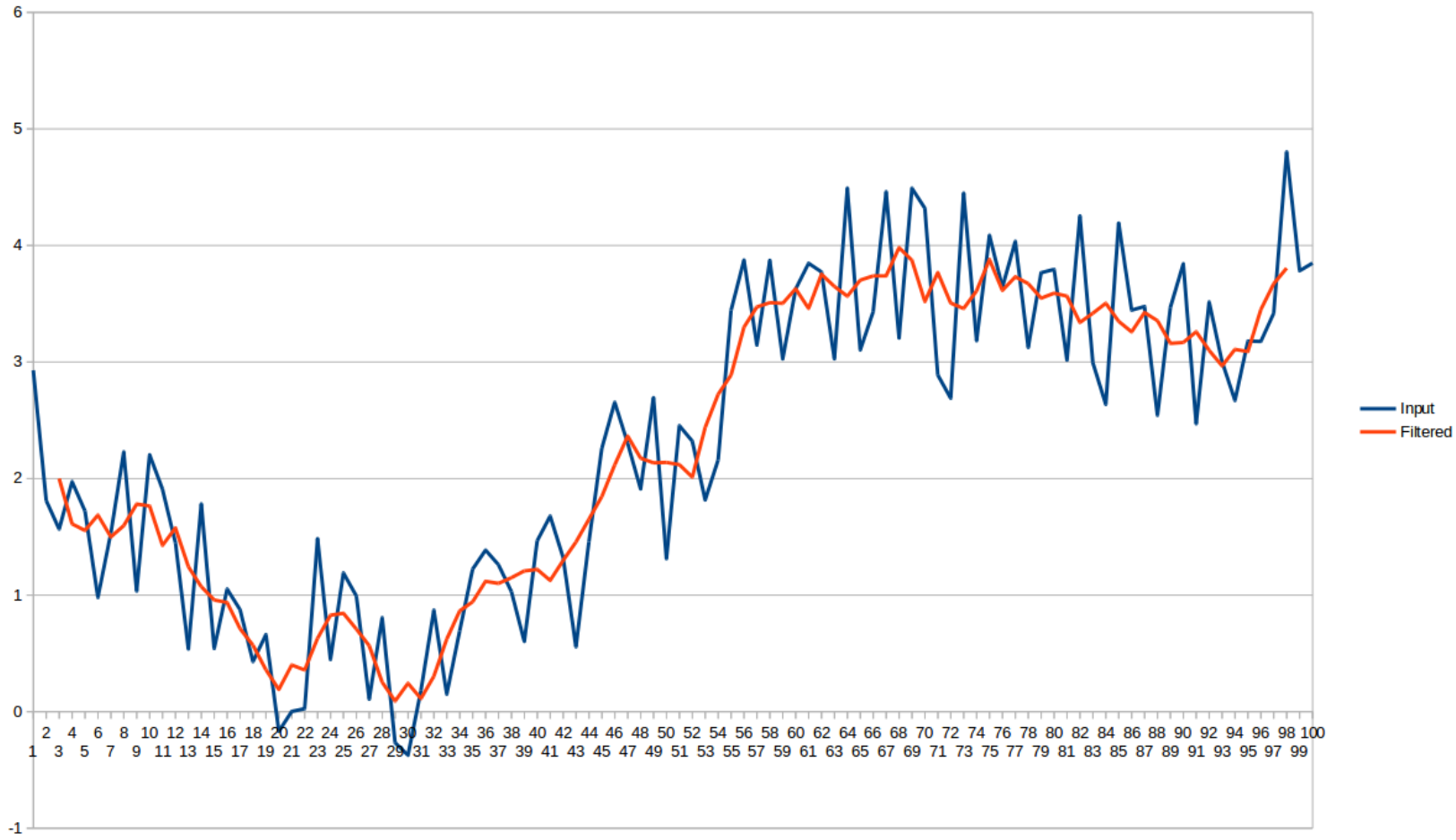
```
data Stream a where
   Stream :: (s -> (a,s)) -> s -> Stream a
```

Compositional list-like interface

```
mapStream :: (a -> b) -> Stream a -> Stream b
mapStream f (Stream next i) = Stream next' i
  where next' s = case next s of
                    (a,s') -> (f a,s')

zip :: Stream a -> Stream b -> Stream (a,b)
zip (Stream next1 i1) (Stream next2 i2) = Stream next (i1,i2)
  where next (s1,s2) = case (next1 s1,next2 s2) of
                         ((a,s1'),(b,s2')) -> ((a,b),(s1',s'2))
```

- The functional stream representation supports *fusion*
- Simply unfold function definitions
- Allows for compositional programming without performance loss
- This transformation happens as a side-effect of how Feldspar is embedded in Haskell.
  No extra implementation effort needed.

```
mapStream f (mapStream g (Stream next init))

==> { unfolding mapStream }

mapStream f (Stream next' init)
  where next' s = case next s of (a,s') -> (g a,s')

==> { unfolding mapStream }

Stream next'' init
  where next' s = case next s of (a,s') -> (g a,s')
        next'' s = case next' s of (b,s') -> (f b,s')

==> { unfolding next' and evaluation }

Stream next'' init
  where next'' s = case next s of (a,s') -> (f (g a),s')
```


6
5
4
3
2
1
0
-1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
Input
Filtered

```haskell
movingAvg :: Fractional a => Data Int -> Stream a -> Stream a
movingAvg n (Stream step init) = Stream step' init'
  where
    init' = (init,replicate n 0)
    step' (s,window) =
      let (a,s')  = step s
          window' = init window ++ singleton a
      in (avg window, (s',window'))
    avg w = sum (elems w) / fromIntegral n
```

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```

- The sliding `window` copied each iteration
- Costly when `window` grow large

```
1     for (uint32_t v47 = 0; v47 < 32; v47 += 1)
2     {
3       //Code for computing the average of the window elided
4       for (uint32_t v89 = 0; v89 < v184; v89 += 1)
5       {
6         ((v46).member2).member2[(v63 + 1)] =
7           ((v48).member2).member2[v63];
8       }
9       //Code updating the struct which holds the window elided
10    }
```

```
1   for (uint32_t v47 = 0; v47 < 32; v47 += 1)
2   {
3     //Code for computing the average of the window elided
4     for (uint32_t v89 = 0; v89 < v184; v89 += 1)
5     {
6       ((v46).member2).member2[(v63 + 1)] =
7         ((v48).member2).member2[v63];
8     }
9     //Code updating the struct which holds the window elided
10  }
```

- Line 4-8 is a loop which copies the window

We would like a representation where we can avoid copying the buffer.

Monads to the rescue

```haskell
data Stream a = Stream (M (M a))
```

```
data Stream a = Stream (M (M a))
```

- The monad `M` in Feldspar is similar to `IO` in Haskell
- Provides mutable references and arrays

```haskell
enumFrom :: Data Int -> Stream Int
enumFrom n = Stream $ do
  r <- newRef n
  loop $ do
    i <- readRef r
    writeRef r (i+1)
    return i
```

```
1  enumFrom :: Data Int -> Stream Int
2  enumFrom n = Stream $ do
3    r <- newRef n
4    loop $ do
5      i <- readRef r
6      writeRef r (i+1)
7      return i
```

- Line 3: Initialization
- Lines 5-7: Loop body
- `loop` = `return`

```haskell
data Stream a = Stream (M (M a))
```

- The outer monad performs initialization
- The inner monad is the loop body.

```haskell
mapStream :: (a -> b) -> Stream a -> Stream b
mapStream f (Stream init) = Stream $ do
  body <- init
  loop $ do
    a <- body
    return (f a)

zip :: Stream a -> Stream b -> Stream (a,b)
zip (Stream init1) (Stream init2) = Stream $ do
  next1 <- init1
  next2 <- init2
  loop $ do
    a <- next1
    b <- next2
    return (a,b)
```

Fusion

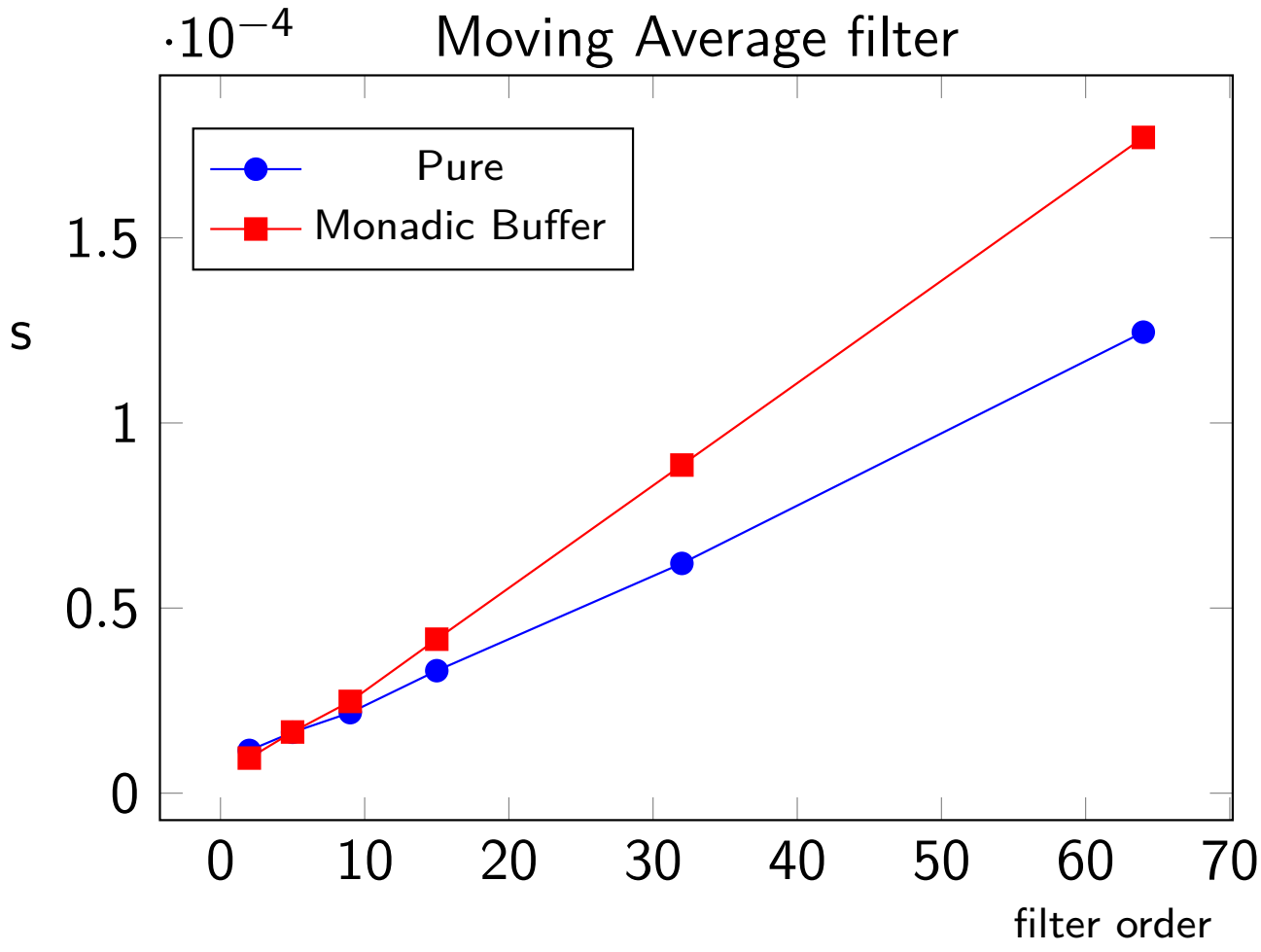
- The new stream type also supports fusion
  - Unfolding of definitions, just like in the functional representation
  - Two of the monad laws: associativity and left identity
- In Feldspar, the two monad laws are also applied as a side-effect of the way it is embedded in Haskell

```haskell
movingAvg :: Int -> Stream Double -> Stream Double
movingAvg n s =
  recurrence (listArray (0,n-1) (replicate n 0.0)) s
             (\input -> sum (elems input) / fromIntegral n)
```

```
recurrence :: Array Int a -> Stream a ->
              (Array Int a -> b) ->
              Stream b
recurrence ii (Stream init) mkExpr = Stream $ do
    next <- init
    ibuf <- initBuffer ii
    loop $ do
      a <- next
      putBuf ibuf a
      b <- withBuf ibuf $ \ib ->
             mkExpr ib
      return b
```

```
1     v14 = 0;
2     copy(v7, zeros);
3     for (uint32_t v24 = 0; v24 < 32; v24 += 1) {
4       v48 = v0[v25];
5       v27 = v14;
6       v14 = ((v27 + 1) % 8);
7       v7[v27] = v48;
8       // Code computing the average elided
9     }
```

- No copying
- Line 7 updates the window


Moving Average filter
$\cdot 10^{-4}$
Pure
Monadic Buffer
s
1.5
1
0.5
0
0
10
20
30
40
50
60
70
filter order

- The cyclic buffer uses **modulus** for each element
- More expensive than copying

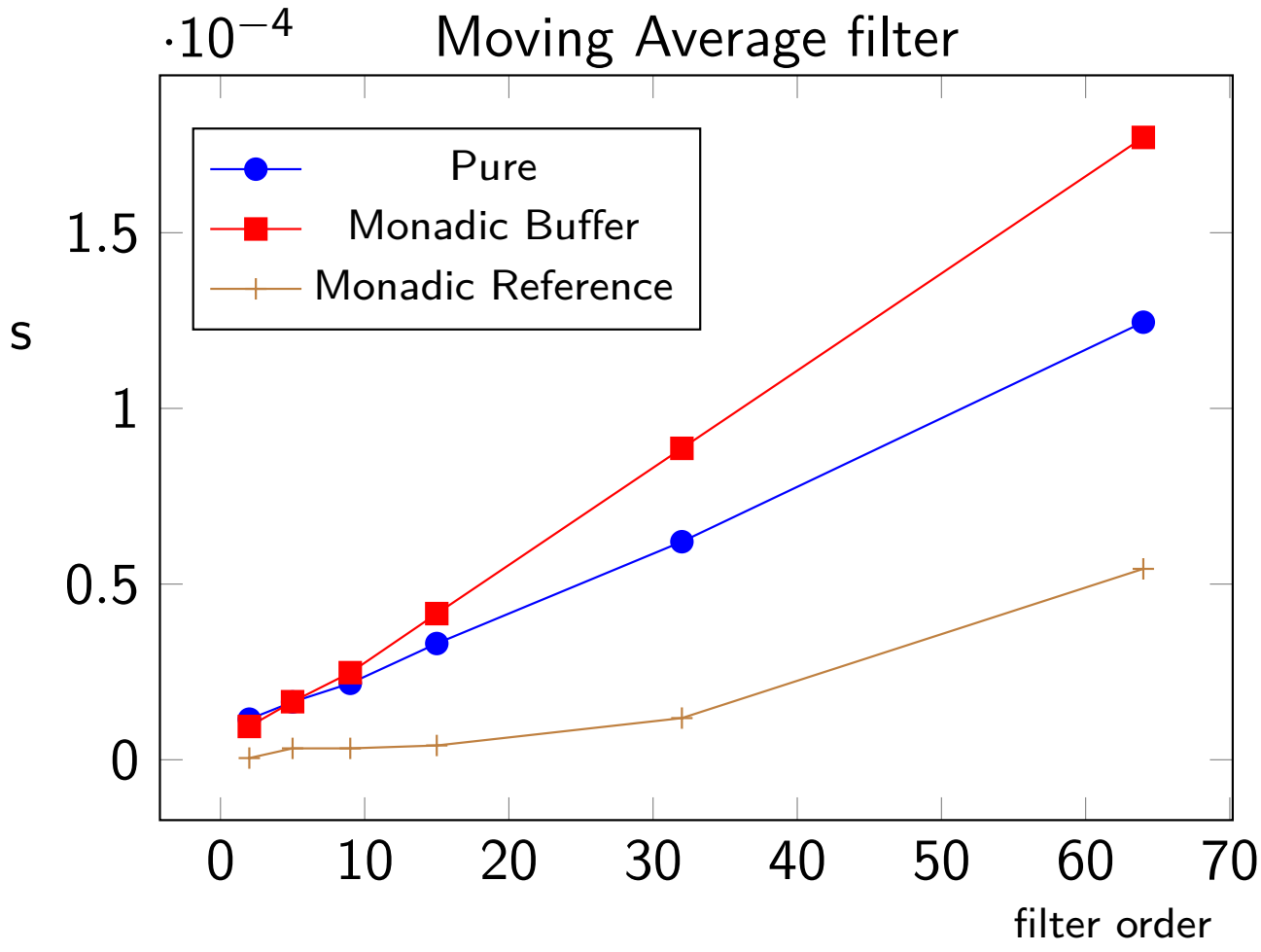
Making a faster filter

Although the mutable buffer was slower, mutable streams allow for many variations.

For instance:

- Unroll the buffer to keep each element in a reference
- Effectively means that each element is kept in a register

```
recurrenceS :: (Type a, Type b) =>
               [Data a] -> Stream (Data a) ->
               ([Data a] -> Data b) ->
               Stream (Data b)
recurrenceS ii (Stream init) mkExpr = Stream $ do
    next <- init
    ris  <- mapM newRef ii
    loop $ do
      a <- next
      if (not $ null ii) then pBuf ris a else return ()
      b <- wBuf ris $ \ib ->
             return $ mkExpr ib
      return b
  where
    pBuf rs a = zipWithM (\r1 r2 -> getRef r1 >>= setRef r2)
                         (tail $ reverse rs) (reverse rs)
             >> setRef (head rs) a
    wBuf rs f = mapM getRef rs >>= f
```


Moving Average filter
$\cdot 10^{-4}$
Pure
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s
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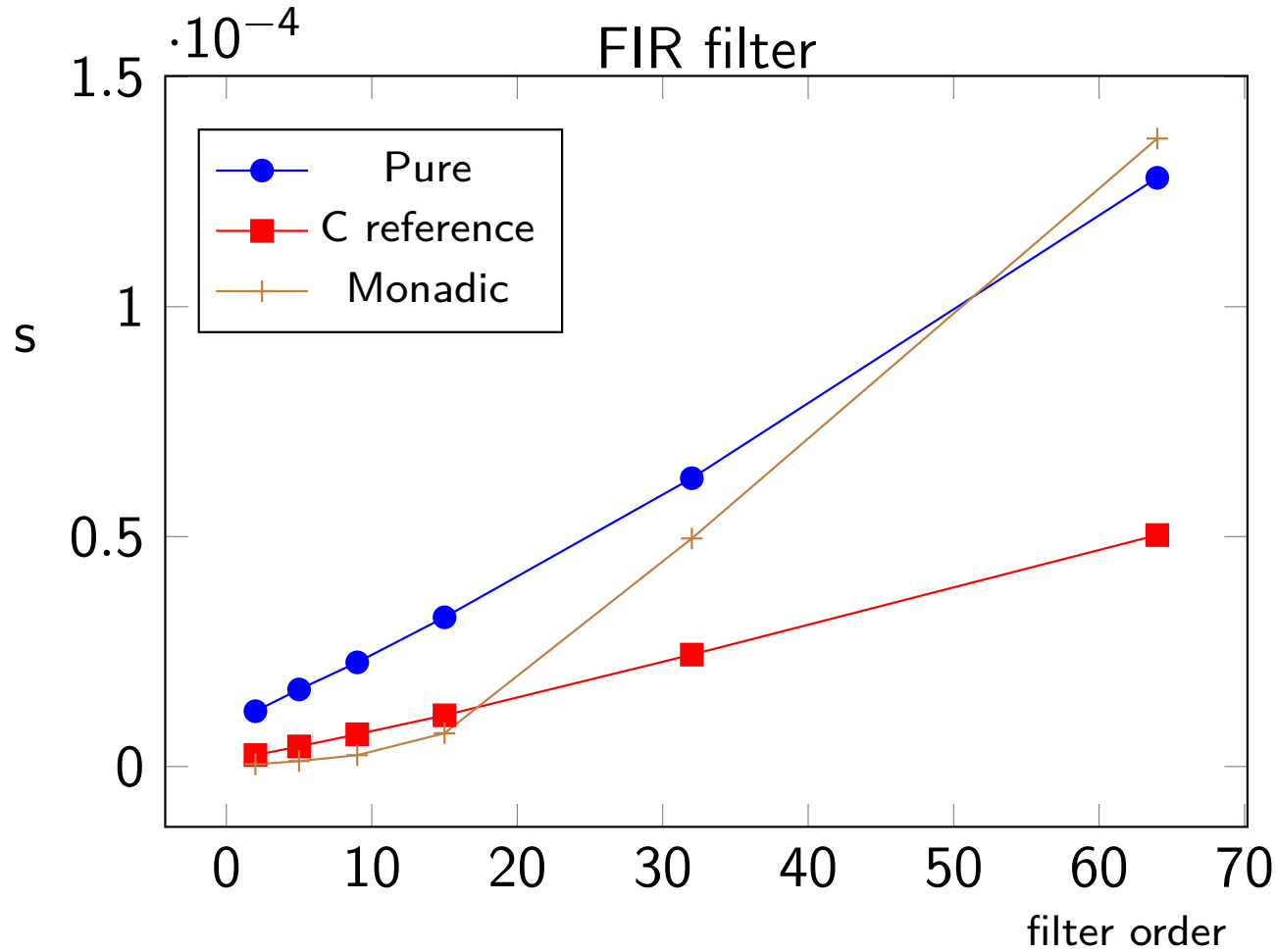
filter order


FIR filter
$\cdot 10^{-4}$
1.5
1
0.5
0
s
Pure
C reference
Monadic
0
10
20
30
40
50
60
70
filter order

Conclusion

- A simple, versatile stream representation
- Enables mutation for efficiency
- A declarative, compositional interface
- Well suited for embedded languages
- Fusion can happen for free with the right embedding

Unexpectedly

- Mutation is not always faster

In the paper

- Avoiding multiple loop variables